Lecture 2:
Preferences and Demand

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Please do not circulate

Announcements

- First lecture for writing next Monday, Feb 2nd.
  - You will see our Writing Assistant
- HW 1 posted
  - Due: Feb 6, Friday, at 11:59 pm CT.

Plan Today

- Review
- Preferences and Demand
  - Utility Function and Marginal Utility
  - Marginal Rate of Substitution (MRS)
  - Utility Miximization

Review

An Introduction to Economic Models

What's a "model" in economics/trade?

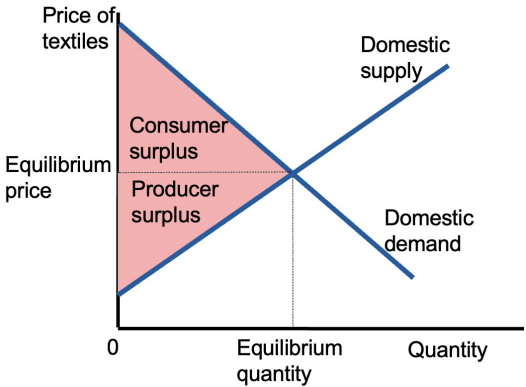
- A model = a map from assumptions to outcomes.
  - Exogenous (given): What we take as fixed—e.g., technologies, factor supplies, preferences, trade costs, policies.
  - Endogenous (determined in model): Prices, wages, quantities produced/consumed, trade flows, etc.
  - Equilibrium concept: The rules that pin down the endogenous objects—e.g., markets clear, firms maximize profits, households maximize utility subject to budget constraints.

We describe every model using the following statement:

- **Given [exogenous things], equilibrium is [endogenous variables] such that [conditions: market-clearing, optimality, feasibility] hold**.

Example from ECON 1101: Supply & Demand - No Trade

- What are the three parts of that model?
- Exogenous:
- Endogenous:
- Equilibrium:
- How do you describe this model?



Example from ECON 1101: Supply & Demand - No Trade

- What are the three parts of that model?
- Exogenous:
  - The S and D curves (shift with costs, income).
- Endogenous:
  - Price & quantity.
- Equilibrium:
  - Price and quantity where $Q^D = Q^S$.
  - For a given supply curve and demand curve, equilibrium is defined by a price and quantity such that demand equals supply.

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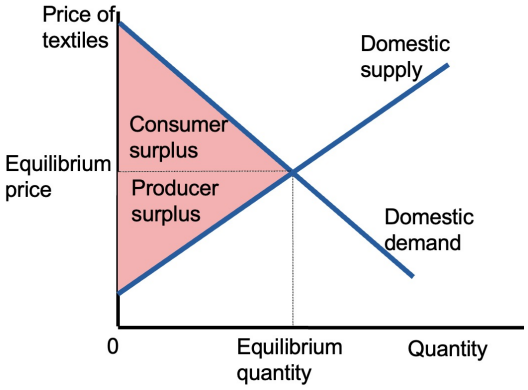
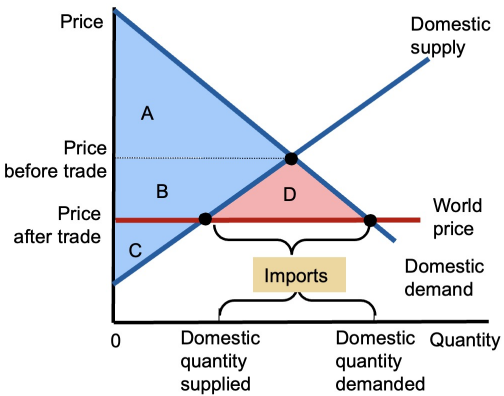
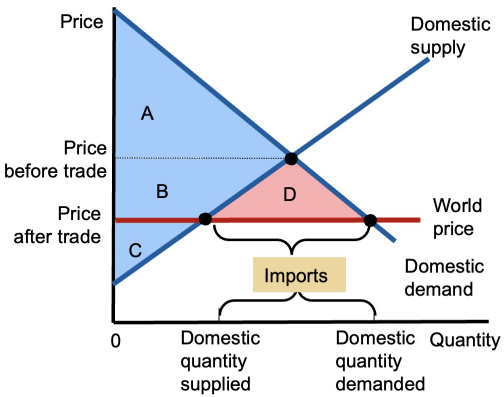


From Domestic S-D to Trade: A Given Lower World Price



- How does the consumer surplus change after opening to trade?
- Producer surplus?
- Total surplus?
- Who are the winners and losers from trade in this world?

From Domestic S-D to Trade: A Given Lower World Price



After trade:

- Consumer surplus
  - changes from A to A+B+D $\Rightarrow$ +(B+D)
- Producer surplus
  - changes from B+C to C $\Rightarrow$ -B
- Total surplus
  - changes from A+B+C to A+B+C+D $\Rightarrow$ +D

Preferences and Demand

How we learn models?

- Intuitions, examples, and maybe graphs first;
- Based on the ideas and examples, we try to define things formally using mathematics;
- This is a general plan for the course: first, provide the intuition, then formalize it. Maybe graphs after math.
- Why are equations helpful?
  - Generalization, not just a particular example or a vague idea;
  - Formal and rigorous;
  - Convenient and organized when dealing with many aspects;
  - Use modern math and computational tools to get quantitative results.
- Does that mean the intuitions are not important?
  - Definitely not!

How we learn models? Cont.

- Go slowly and step by step.
- Slow me down if I go too fast.
- Lecture slides cannot have all the contents.
- Whiteboard and verbal explanations to further illustrate the concepts and models.
- Your participation in class matters a lot.
- Ask questions whenever you have one.

Environment and Endowment

We begin with a single country, two goods indexed by $i \in \{1, 2\}$, and a single consumer. A consumption bundle is (C_1, C_2) with $C_i \geq 0$. Prices are (p_1, p_2) with $p_i > 0$.

- Two goods: C_1 and C_2.
- Prices: p_1 and p_2.
- No trade yet: we use this partial-equilibrium consumer side to prepare for general equilibrium (GE) trade models.

Endowments:
The consumer is endowed with $E = (E_1, E_2)$ units of the two goods. At prices (p_1, p_2), the value/income of the endowment is

$$I \;=\; p_1 E_1 + p_2 E_2.$$

I is the total income the consumer can get from this endowment economy which limits total spending.

Budget Set

Given prices (p_1, p_2) and income I, the affordable set is

$$\mathcal{B}(p_1, p_2; I) \;=\; \{(C_1, C_2) \,:\, p_1 C_1 + p_2 C_2 \leq I,\; C_1 \geq 0,\; C_2 \geq 0\}.$$

The budget line/constraint is

$$p_1 C_1 + p_2 C_2 = I.$$

Its intercepts are I/p_1 on the C_1-axis and I/p_2 on the C_2-axis; its slope is $-p_1/p_2$ (the relative price).

Graphically (next slide)...

Budget Set - Graphic

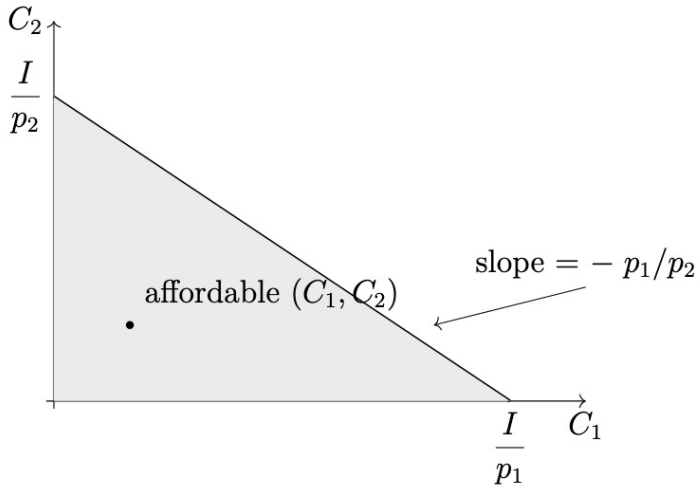


Figure: Budget set at prices (p_1, p_2) and income I: $\{(C_1, C_2) : p_1 C_1 + p_2 C_2 \leq I,\ C_1, C_2 \geq 0\}$.

Budget Set with Endowment

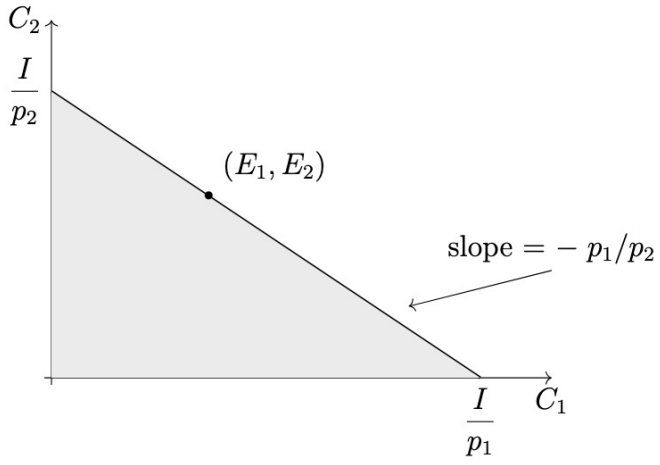


Figure: Budget set with endowment: the budget line $p_1 C_1 + p_2 C_2 = p_1 E_1 + p_2 E_2$ passes through (E_1, E_2) and has slope $-p_1/p_2$.

Preference and Utility Function

Utility function:
Preferences are represented by a utility function $U(C_1, C_2)$.

- Utility is ordinal: only the ranking over consumption bundles matters
- any strictly increasing transformation (e.g., $\log U$, U^2, etc.) represents the same preferences, and units don't matter.

Indifference-curve:
For any certain level of utility $\bar{U}$, the set of consumption bundles $\{(C_1, C_2) : U(C_1, C_2) = \bar{U}\}$ is an indifference curve.

- All points (consumption bundles) on the same curve give the same level of utility.
- Higher curves correspond to higher welfare.

Indifference Curves - Graphic

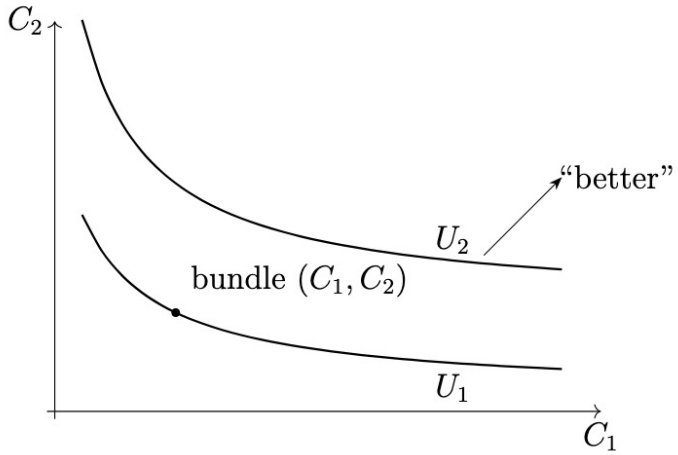


Figure: Indifference-curve map: higher curves (U_2) represent preferred bundles relative to lower curves (U_1).

Marginal Utility

The definition of marginal utility is given by

$$MU_1(C_1, C_2) \equiv \frac{\partial U(C_1, C_2)}{\partial C_1} \geq 0, \qquad MU_2(C_1, C_2) \equiv \frac{\partial U(C_1, C_2)}{\partial C_2} \geq 0.$$

What "marginal utility" means?

- At a bundle (C_1, C_2), the marginal utility of good i is the instantaneous rate at which utility changes when you increase C_i slightly, holding the other good fixed.
- It is a local concept: it tells you the slope of the utility function $U(\cdot)$ in the C_i direction at the current bundle.
- Throughout, we assume marginal utilities are non-negative, so "a little more" of any good weakly increases utility.

Marginal Utility - Derivative-based Intuition

Formally, for a small change in any consumption $h > 0$,

$$MU_1(C_1, C_2) = \lim_{h \to 0} \frac{U(C_1 + h,\ C_2) - U(C_1,\ C_2)}{h},$$
$$MU_2(C_1, C_2) = \lim_{h \to 0} \frac{U(C_1,\ C_2 + h) - U(C_1,\ C_2)}{h}.$$

Remark - units and interpretation:
MU_i has units of "utils per unit of good i," but because utility is ordinal, the absolute levels of MU_i are not meaningful; signs, differences, and ratios (e.g., MU_1/MU_2 for MRS below) are what will matter.

Marginal Rate of Substitution (MRS)

Definition:
Fix a utility level $\bar{U}$ and consider an indifference curve $\{(C_1, C_2) : U(C_1, C_2) = \bar{U}\}$. At any (differentiable) point (C_1, C_2) on this curve, the marginal rate of substitution is

$$\mathrm{MRS}(C_1, C_2) \equiv \frac{MU_1(C_1, C_2)}{MU_2(C_1, C_2)} \geq 0, \quad \text{where} \quad MU_i \equiv \frac{\partial U}{\partial C_i}, \ \forall\, i \in \{1, 2\}.$$

We will always refer to MRS in absolute value (a positive number).

Interpretation: (units of good 2 per unit of good 1)
$\mathrm{MRS}(C_1, C_2)$ is the amount of good 2 the consumer is willing to give up to obtain one more unit of good 1 while staying on the same indifference curve (same utility).
It captures the local substitutability between the two goods at the current bundle.

Marginal Rate of Substitution (MRS) - Graphic

Graphically, we have

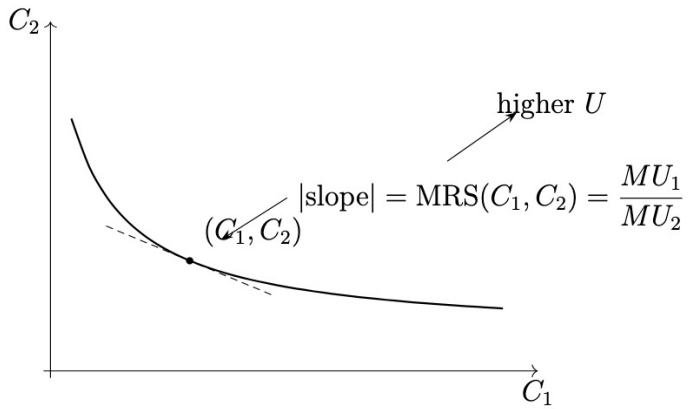


Figure: At (C_1, C_2), the absolute slope of the indifference curve equals $\text{MRS} = MU_1/MU_2$.

Convex (weakly) preferences

Definition:

Preferences are (weakly) convex if for any two consumption bundles A and B with $A \sim B$ (indifferent), all convex combinations $\lambda A + (1 - \lambda)B$ with $\lambda \in [0, 1]$ are (weakly) preferred to A (and B).

Graphically, indifference curves for convex (weakly) preferences are bowed toward the origin.

Diminishing MRS under convexity

Definition:

If preferences are (weakly) convex and differentiable with marginal utility $MU_i \geq 0$, then along a given indifference curve, $\mathrm{MRS}(C_1, C_2)$ decreases as C_1 increases (and C_2 falls).

Intuition:

The more of good 1 you already have, the less of good 2 you are willing to give up for one more unit of good 1.

Graphically (next slide),

Diminishing MRS under convexity - Graphic

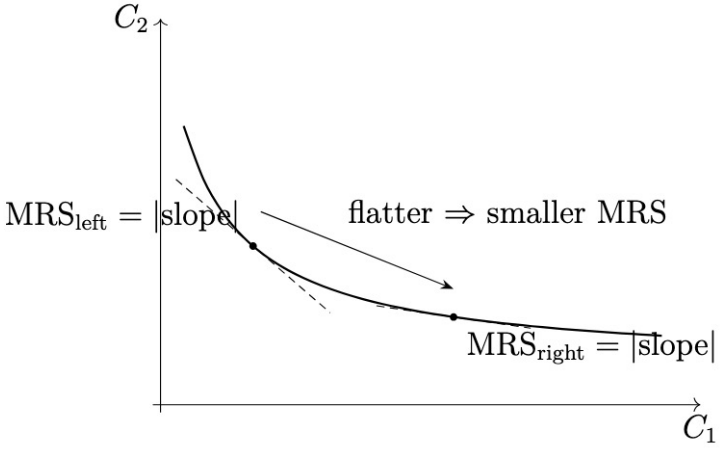


Figure: Convex preferences $\Rightarrow$ indifference curves bowed to origin and diminishing MRS as C_1 increases.

Example: Linear Preferences

Definition: Linear (perfect-substitutes) preferences
Preferences are linear if and only if their utility representation can be expressed by

$$U(C_1, C_2) \;=\; a\,C_1 + b\,C_2, \qquad a \geq 0,\; b \geq 0,\; (a, b) \neq (0, 0).$$

Marginal utilities are constant and non-negative:

$$MU_1 = a \geq 0, \qquad MU_2 = b \geq 0,$$

and the marginal rate of substitution is constant:

$$\mathrm{MRS}(C_1, C_2) \;=\; \frac{MU_1}{MU_2} \;=\; \frac{a}{b}.$$

Indifference curves are straight lines with slope $-\frac{a}{b}$.
There is no diminishing MRS (can you show that from a graph?).

Example: Leontief Preferences

Definition: Leontief (perfect-complements) preferences
Preferences are called Leontief if its utility representation can be expressed by

$$U(C_1, C_2) \;=\; \min\{\alpha\, C_1,\ \beta\, C_2\}, \qquad \alpha > 0,\ \beta > 0.$$

Goods are used in fixed proportion:

$$\beta\, C_2 = \alpha\, C_1 \quad \Longleftrightarrow \quad C_2 = \frac{\alpha}{\beta}\, C_1.$$

Indifference curves are L-shaped with kink at $(\bar{U}/\alpha,\ \bar{U}/\beta)$ for utility level $\bar{U}$. MRS is undefined since at the kink $(\bar{U}/\alpha,\ \bar{U}/\beta)$ the indifference curve is not differentiable.

Question:

- Can you think of some real-world examples of Leontief preferences?
- Graph?

Utility Maximization

Utility maximization problem: setup and optimality conditions
Given prices $p = (p_1, p_2) > 0$ and income $I > 0$, the consumer chooses $(C_1, C_2) \in \mathbb{R}^2_+$ to

$$\max_{C_1, C_2 \geq 0} \; U(C_1, C_2) \quad \text{s.t.} \quad p_1 C_1 + p_2 C_2 \leq I.$$

Throughout, assume U is increasing (so marginal utilities are non-negative) and differentiable on $\mathbb{R}^2_{++}$.

Graphically, we do we see?

Utility Maximization - Lagrangian methods

Let $\lambda \geq 0$ be the multiplier on the budget constraint, and $\mu_1, \mu_2 \geq 0$ on $C_1 \geq 0$, $C_2 \geq 0$. The Lagrangian is

$$\mathcal{L} = U(C_1, C_2) + \lambda (I - p_1 C_1 - p_2 C_2) + \mu_1 C_1 + \mu_2 C_2.$$

First-order and complementary-slackness conditions:

$$\frac{\partial \mathcal{L}}{\partial C_1} = MU_1 - \lambda p_1 + \mu_1 = 0, \qquad \frac{\partial \mathcal{L}}{\partial C_2} = MU_2 - \lambda p_2 + \mu_2 = 0,$$
$$\lambda \geq 0, \quad I - p_1 C_1 - p_2 C_2 \geq 0, \quad \lambda (I - p_1 C_1 - p_2 C_2) = 0,$$
$$\mu_i \geq 0, \quad C_i \geq 0, \quad \mu_i C_i = 0 \quad (i = 1, 2).$$

Utility Maximization - KKT vs Complementary Slackness

We need to look at what the KKT conditions are and what the complementary slackness conditions are in our consumer utility maximization problem (not required for this class).

Stationarity (KKT):

$$\frac{\partial \mathcal{L}}{\partial C_1} = MU_1 - \lambda p_1 + \mu_1 = 0, \qquad \frac{\partial \mathcal{L}}{\partial C_2} = MU_2 - \lambda p_2 + \mu_2 = 0.$$

Primal feasibility (KKT):

$$C_1 \geq 0, \quad C_2 \geq 0, \quad I - p_1 C_1 - p_2 C_2 \geq 0.$$

Dual feasibility (KKT):

$$\lambda \geq 0, \quad \mu_1 \geq 0, \quad \mu_2 \geq 0.$$

Complementary slackness (part of KKT):

$$\lambda \left(I - p_1 C_1 - p_2 C_2\right) = 0, \qquad \mu_1 \, C_1 = 0, \qquad \mu_2 \, C_2 = 0.$$

Next Lecture

- Utility Maximization and Lagrangian Methods (continued)
- Relative Demand
- Homothetic Preference
- Gravity Equation

Thank you!